Graphing Challenge: A Novel Twist on a Standard Idea from Algebra

Graphs and Equations on Non-Standard Axes

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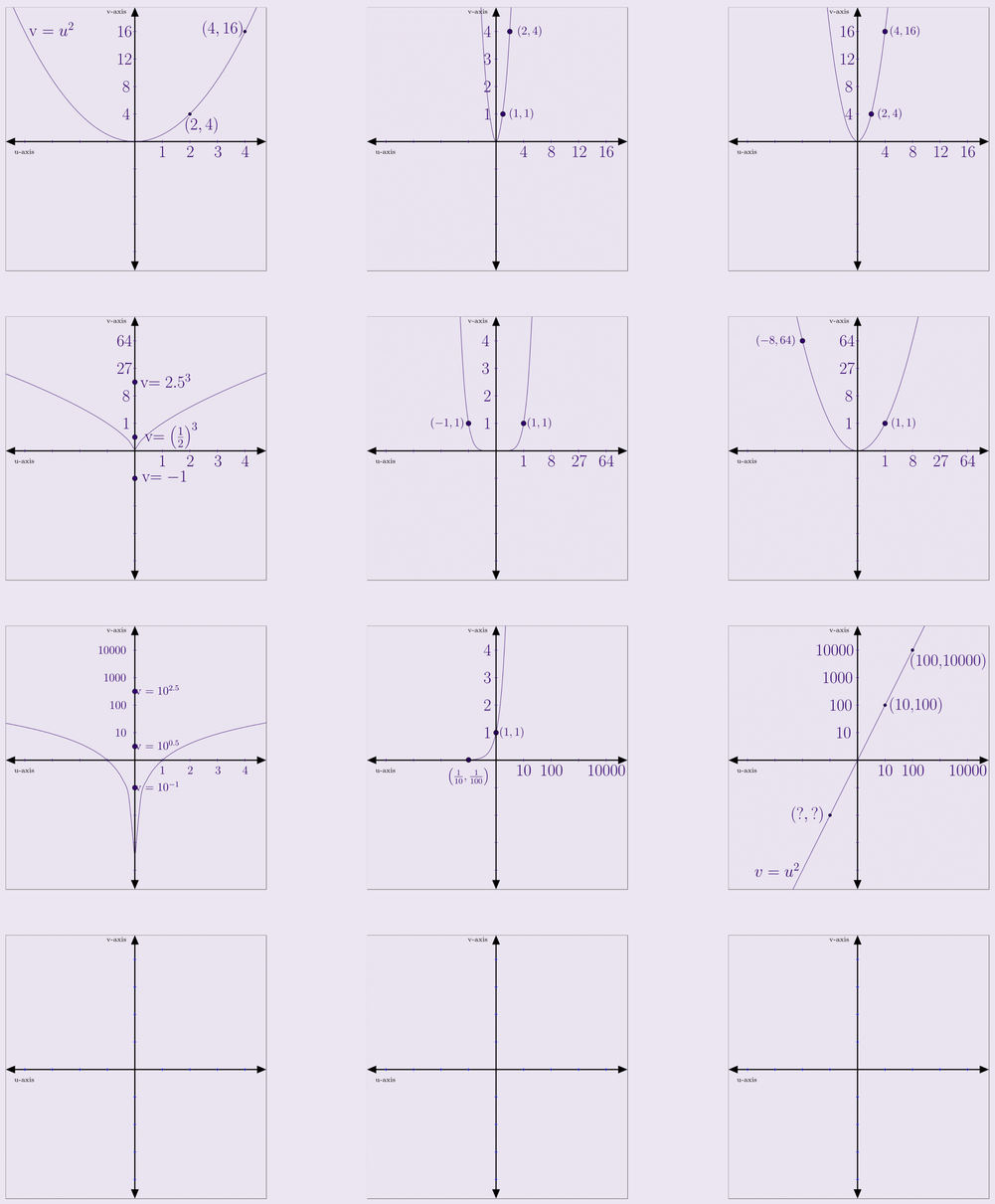
Exploration 1: Graphing

Your Mission:

Plot $v = u^2$ on each!

Lift the flaps to see solutions!



Tips for Exploration 1

Tip 1: Start by labelling the unmarked notches. The ones that go 10, 100, 1000 are worth thinking twice about. What comes before 10 in that pattern? And what's going on in between 10 and 100?

Tip 2: Try plotting $(1, 1)$, $(2, 4)$, and $(3, 9)$ on each to get started. Can you find $\left(\frac{1}{2}, \frac{1}{4}\right)$? Can you find $(-1, 1)$? What about $(100, 10000)$?

Tip 3: The last row is intentionally unmarked. Label it as you please!

Learning Objectives

- **Students exercise their own curiosity and inquiry skills** by creating and analyzing their own coordinate systems.
- **Students practice flexible mathematical thinking** as they explore uncharted territory.
- **Students learn to distinguish** between an equation and its graph, and they learn that the appearance of its graph depends on the choice of coordinate system.
- **Students gain familiarity** with less common graphing systems, such as log-log and log-lin.

Common Core Practices and Content Standards

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
- HSA-REI.D.10-11, HSA-CED.A.2, HSF-BF.A.1, HSF-IF.B.5

Mathematical Context

In the process of these explorations, students have an opportunity to see a huge swath of connections. Ideas of inverse functions and relationships between exponential functions, logarithmic functions, parabolas, and lines come to life in an open-ended and accessible way, that **piques genuine student curiosity**.

There's lots to discover by playing around here–these ideas are very much related to some topics that students normally wouldn't see until a course in linear algebra or multivariable calculus.

Exploration 2: Modeling

Exploration 1 demonstrates that the graph of an equation can look quite different depending on your choice of coordinate system. Sometimes, these graphs look quite foreign. But other times, they're quite familiar: one of the examples looks just like the familiar line we think of as $y = 2x$. Looking closer at that particular coordinate system, three equations stand out that help relate x, y, u, and v:

$$v = u^2$$
$$v = 10^y$$
$$u = 10^x$$

Can you see how these three equations lead to the conclusion that on this coordinate system, the graph of $v = u^2$ will remind us of the more familiar equation $y = 2x$?

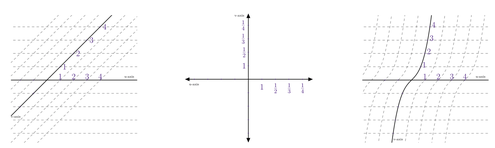
Your Mission:

Find equations in terms of x and y to match each of the plots from Exploration 1.

Tip: For each coordinate system, write down the three key equations relating x, y, u, and v.

Extensions

Here's a few trickier coordinate systems, for those who are feeling ambitious:



Classroom-ready handouts and solutions are available at www.PaulGafni.com/Graphing

For questions, suggestions, or feedback, contact Paul at gafnip@uw.edu.